Proof Rules(Natural Deduction)

1. Which rule of inference is used in the following arguments:

   a.)“All men are mortal. Socrates is a man. Therefore, Socrates is mortal.”

   (**HINT:** Universal instantiation is used to conclude that “If Socrates is a man, then Socrates is mortal.” Modus ponens is then used to conclude that Socrates is mortal.

   b.) Jerry is a mathematics major and a computer science major. Therefore, Jerry is a mathematics major.

   (HINT: Simplification)

   c.) “Randy works hard,” “If Randy works hard, then he is a dull boy,” and “If Randy is a dull boy, then he will not get the job” imply the conclusion “Randy will not get the job.”

   (**HINT:** Let w be “Randy works hard,” let d be “Randy is a dull boy,” and let j be “Randy will get the job.” The hypotheses are $w, w \rightarrow d$, and $d \rightarrow \neg j$. Using modus ponens and the first two hypotheses, d follows. Using modus ponens and the last hypothesis, $\neg j$, which is the desired conclusion, “Randy will not get the job,” follows.)

   d.) "If it is rainy, then the pool will be closed. It is rainy. Therefore, the pool is closed."

   (**Hint:** Modus Ponens).

   e.) "If it snows today, the university will close. The university is not closed today. Therefore, it did not snow today." (**Hint:** Modus tollens).

2. For each of these collections of premises, what relevant conclusion or conclusions can be drawn?

   a) “If I take the day off, it either rains or snows.” “I took Tuesday off or I took Thursday off.” “It was sunny on Tuesday.” “It did not snow on Thursday.”

   (**HINT:** Valid conclusions are “I did not take Tuesday off,” “I took Thursday off,” “It rained on Thursday.”)

   b) “If I eat spicy foods, then I have strange dreams.” “I have strange dreams if there is thunder while I sleep.” “I did not have strange dreams.”

   (**HINT:** “I did not eat spicy foods and it did not thunder” is a valid conclusion.)

   c) “I am either clever or lucky.” “I am not lucky.” “If I am lucky, then I will win the lottery.”

   (**HINT:** “I am clever” is a valid conclusion.)

3. Show that if $\forall x(P(x) \rightarrow (Q(x) \wedge S(x)))$ and $\forall x(P(x) \wedge R(x))$ are true, then $\forall x(R(x) \wedge S(x))$ is true.

(**HINT:** Use rule of inference.)

4. For each of these arguments determine whether the argument is correct or incorrect and explain why?

   a) All students in this class understand logic. Xavier is a student in this class. Therefore, Xavier understands logic.

   (**HINT:** Correct, using universal instantiation and modus ponens.)

   b) Every computer science major takes discrete mathematics. Natasha is taking discrete mathematics. Therefore, Natasha is a computer science major.

   (**HINT:** Invalid; fallacy of affirming the conclusion.)

   c) All parrots like fruit. My pet bird is not a parrot. Therefore, my pet bird does not like fruit.

   (**HINT:** Invalid; fallacy of denying the hypothesis.)

   d) Everyone who eats granola every day is healthy. Linda is not healthy. Therefore, Linda does not eat granola every day.

   (**HINT:** Correct, using universal instantiation and modus tollens.**)**

5. For each of these collections of premises, what relevant conclusion or conclusions can be drawn?

   a.) "Every computer science major has a personal computer." "Ralph does not have a personal computer." "Ann has a personal computer."

   (**HINT:** "Ralph is not a CS major" is a valid conclusion)

   b.) "What is good for corporations is good for the United States." "What is good for the United States is good for you." "What is good for corporations is for you to buy lots of stuff."

   (**HINT:** "That you buy lots of stuff is good for the U.S. and is good for you" is a valid conclusion.)

   c.) "All rodents gnaw their food." "Mice are rodents." "Rabbits do not gnaw their food." "Bats are not rodents.

   (**HINT:** "Mice gnaw their food" and "Rabbits are not rodents" are valid conclusions.)

6. Proof :

   a.) $p, \neg\neg(q \land r) \vdash \neg\neg p \land r$

   b.) $p \land q, r \vdash q \land r.$

   c.) $p \rightarrow (q \rightarrow r), p, \neg r \vdash \neg q$

   d.) $p \land q \rightarrow r \vdash p \rightarrow (q \rightarrow r)$

(**HINT:** Use proof rules.)

7. Prove that the sequence $p \rightarrow (q \rightarrow r), p, \neg r \vdash \neg q$ is valid, without using the MT rule.
8. What rule of inference are used in this argument?
   "No man is an island. Manhattan is an island. Therefore, Manhattan is not a man."

9. Show that the premises "Everyone in this discrete mathematics class has taken a course in computer science" and "Marla is a student in this class" imply the conclusion "Marla has taken a course in computer science."
   (**HINT:** Using Inference Rules for quantified statements.)

10. Show that the premises "A student in this class has not read the book," and "Everyone in this class passed the first exam" imply the conclusion "Someone who passed the first exam has not read the book."
    (**HINT:** Using Inference Rules for quantified statements.)

RESOURCES :

1) Logic in Computer Science, Huth and Ryan.
2) Discrete Mathematics and Its application, Kenneth Rosen.
3) http://www.iis.sinica.edu.tw/~bywang/courses/comp-logic/chapter-1-1.pdf